The Earth's Energy Balance

There are three modes of energy transfer

- Conduction – molecular interactions
- Convection – fluid motion
- Radiation - self propagating

Fundamental Equations

All radiation propagates at a constant speed through a vacum. This is the speed of light (c) and is equal to 3 x 10^8 m sec^{-1}.

$$c = \nu \lambda$$

Where υ is the frequency and λ is the wavelength.

Studying the particle nature electromagnetic radiation, it was determined that the energy of an individual photon is quantized, that is to say is it has can have specific values (often referred to as states). The energy of a photon is:

$$E_p = nh\nu = nh\frac{c}{\lambda}$$

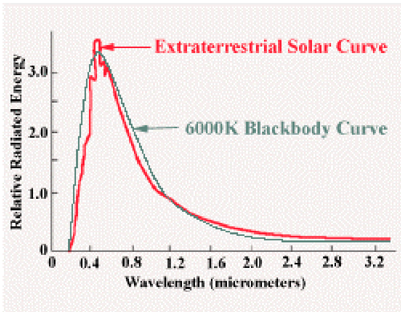
Where *n* is an integer (1,2,3...) and *h* is Plank's constant (6.626x10^{-34} J sec). From the above equation it is possible to see that the energy of a photon is inversely related to its wavelength.

All objects with absolute temperatures above 0° K emit radiation

In ~1900, Max Planck, determined the spectral emmittance (S_λ) of energy, that is too say how much energy is emitted at a specific wavelength from an object with a certain absolute temperature (*T* in Kelvins)

$$S_\lambda = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/kT\lambda} - 1}$$

Where k is the Boltzman constant ($1.3805x10^{-23}$ JK^{-1}).



If we differentiate the formula above, and then set the differential to zero, we can find the wavelength of maximum emission. Differentiation, simply yields the slope at each point on the Planck curve and at the point of maximum emission, the slope will go to zero, that is at this maximum point, the slope is flat. This differentiation yields:

$$\lambda_{\max} = \left(2.90x10^{-3} m \cdot K\right)\frac{1}{T}$$

And is know as *Wein's Displacement Law*. This law says that the higher the temperature of an object, the shorter the wavelength of maximum emmission. The equation can be rearranged to determine the *color temperature (T_c)*.

$$T_c = \frac{2.90x10^{-3} m \cdot K}{\lambda_{\max}}$$

For an example let us use the sun. The sun has a maximum wavelength of emission of 0.475 µm or 0.000000475 m. This is in the blue portion of the spectra. This leads to a color temperature of approximately 6100° K. Now let us compare this to a standard incandescent light bulb. The temperature of the filament of a standard light bulb is much cooler than the sun's color temperature (~2170C). Using *Wein's Displacement Law*, the wavelength of maximum emission would be 1.3 µm).

One very important property is not how much energy a body emits at a certain wavelength, but how much energy it emits at all wavelengths. To determine this it is necessary to integrate the Planck function. Integration in this case means determining the area that lies under the curve in **Figure 1**.

$$S = \int S_\lambda d_\lambda = \int \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/kT\lambda} - 1} d_\lambda$$

$$S = \sigma T^4$$

The above equation is known as the *Stefan-Boltzman* law and σ is known appropriately as the Stefan-Boltzman constant ($5.67\text{x}10^{-8}$ W m^{-2} K^{-4}).

Until now we have assumed that any body with an absolute temperature above 0° K is emitting radiation perfectly. Such objects are known as blackbodies. However, most objects are not perfect emitters, but in fact emit radiation with less than 100% efficiency. These objects are known as graybodies. To account for this behavior a factor known as emmissivity (ε) is added to the Stefan-Boltzman equation. ε is a dimensionless, and has a value between 0 and 1. In fact, for many objects, ε will vary as a function of wavelength, that is to say that objects emit with varying efficiency as a function of wavelength, just as their reflectance varies as a function of wavelength. Thus the Stefan-Bolzman relationship for graybodies is:

$$S = esT^4$$

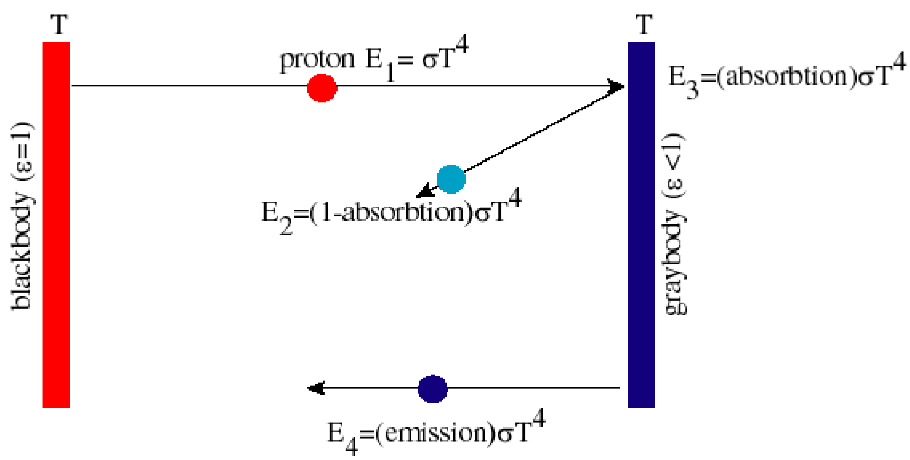
Let us take a look at the possible fate of photons… They can be…

- Absorbed (A)
- Reflected (R)
- Emitted (E)
- And A + R + E = 1

As is shown in Figure 2 below, E = A, which is known as Kirkoff's law.

Kirchoff's Law

at the same temperature, emission equals absorbtion



E1 = E2 + E4
E3 = E1 - E2 or E1 = E3 + E2
therefore E3 = E4

if T(blackbody) = T(graybody) then
absorbtion = emission

Solar Radiation

Let us get a little more practical… .

We all know that the Sun emits energy, and now know that the amount of energy that it emits is a function of its temperature. The amount of energy the earth receives at the top of the atmosphere from the sun is known as the *solar constant*. This energy that the earth receives from the sun is the major source of energy running the earth system. Radioactive decay within the earth is the other source of energy, but is of a much smaller magnitude.

In terms of radiative emission, let us consider the sun with an equivilant blackbody temperature of 5770° K. According to the Stefan-Bolzman Equation, the amount of energy emmitted from the sun is

$$E = \left(5.67x10^{-8} W \cdot m^{-2} \cdot K^{-4}\right)\left(5770K\right)^4 = 6.28x10^8 W \cdot m^{-2}$$

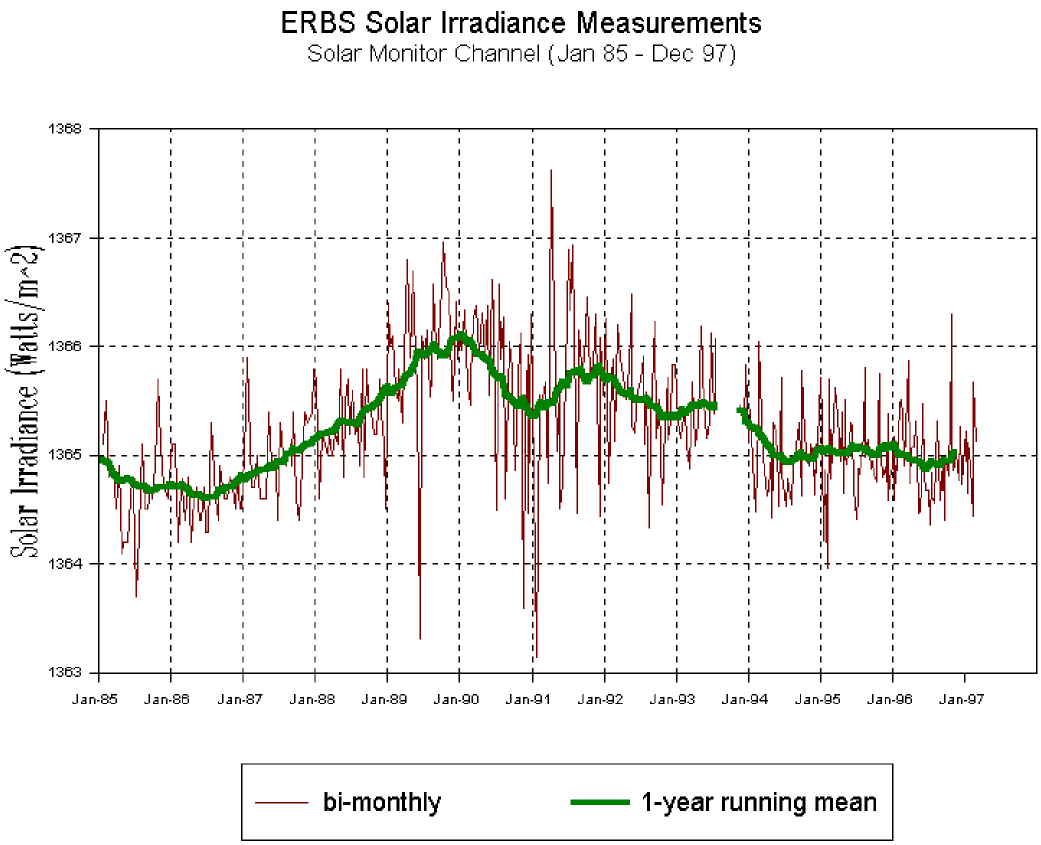
Note that this is the amount of energy emitted from *every* square meter of the sun. To find the total amount of energy emitted from the sun we need to multiply this value by the total surface area of the sun ($4\pi r_{sun}^2$) where the radius of the sun is $7x10^8$ m:

$$E_{sun} = E\left(4pr_{sun}^2\right) = 6.15\mathrm{e}+18\ \mathrm{W}$$

This energy propagates through space, however as the distance away from the sun increases, the amount of energy incident on a square meter area will drop off proportionally to the square of the distance from the sun to the earth is $1.5x10^{11}$m. You might remember this relationship from high school physics. Thus by the time the energy from the sun reaches the earth, the amount incident on a square meter area is:

$$E_{earth} = \frac{E_{sun}}{4p^{\,2}_{r\ sun-earth}} = \frac{6.15x1018W}{4p\left(1.5x10m\right)^2} = 1367W \cdot m^{-2}$$

If we look at Figure 3, we can see how good of a job we actually did. These measurements of the solar constant were made by and instrument aboard the ERBE instrument.



Radiation Balance of the Earth

The net radiation balance of the surface of the earth (Q^*) is the sum of 4 components:

$$Q^* = K\downarrow + K\uparrow + L\downarrow + L\uparrow$$

Where K is the solar (shortwave radiation) from the sun and L is longwave radiation emitted by the atmosphere and the ground. Shortwave radiation refers to radiation at wavelengths between say 0.2 and 3.0 µm (some people extend the range out to ~10µm). This is energy that had its origins with the sun. Longwave radiation is the radiation that is emitted by the earth and atmosphere. Because of their lower temperatures the wavelength of maximum emittance is longer (remember *Wien's Displacement Law*). Thus longwave radiation refers to the 3-100 µm. The arrows indicate the direction the radiation is propagating (down and up). Downward fluxes are considered positive, upward fluxes, negative.

Let us consider the earth as a solitary *blackbody* in space that is in *radiative equilibrium* with the sun, that is to say, the amount of energy it admits is exactly equal to the amount it receives from the sun. This means that it is receiving 1367 Wm-2 of energy from the sun (Kdown). Since it is a blackbody there is no reflection (Kup = 0). Likewise since there is no atmosphere, there is longwave radiation directed toward the ground (Ldown=0). Thus:

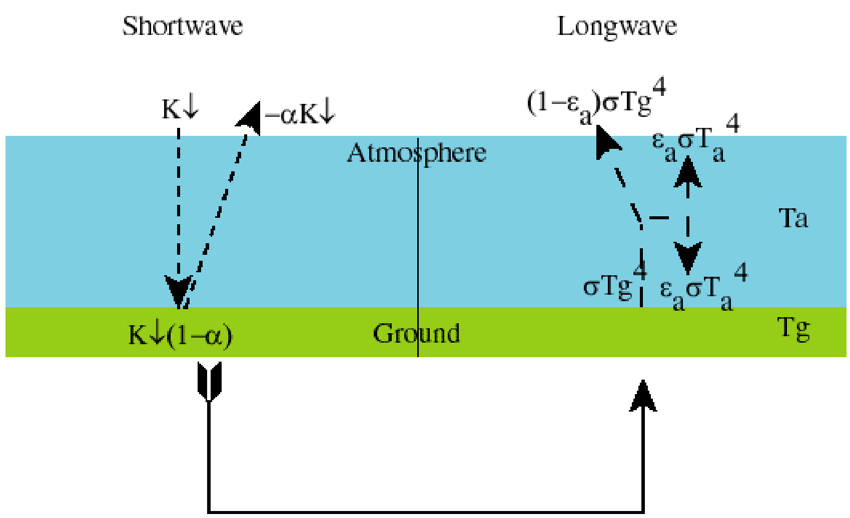
$$Q^{*} = 1367 W \cdot m^{-2} + 0 + 0 - esT^{4}$$

Solving for T (the *radiative equilbrium* temperature), we get 394°K or 121°C.

Also, Albedo can be defined as:

$$a = \frac{K\uparrow}{K\downarrow}$$

Now let us make a 0-dimensional energy balance model of the earth.



Let us assume that the Ground and the Atmosphere have the following properties:

		Shortwave	**Longwave**
Ground (g)	Reflection	α	0
	Absorption	(1-α)	1
	Transmission	0	0
Atmosphere (a)	Reflection	0	0
	Absorption	0	εa
	Transmission	1	εa

With these quantities, the following equations hold:

At the top of the Atmosphere:

$$Q^* = K\downarrow - aK\downarrow + 0 - \left[(1-e_a)sT_g^4 + e_a sT_a^4\right]$$

At the ground Surface:

$$Q^* = K\downarrow - aK\downarrow + 0 - sT_g^4 + e_a sT_a^4$$

Solving these equations simultaneously we achieve the following:

$$T_g = \sqrt[4]{\frac{K\downarrow(1-a)}{s\left(1-\frac{e_a}{2}\right)}}$$

Before we 'run' our zero-dimensional energy balance model of the earth (the above equation), we need to determine the average amount of energy the earth receives from the sun over the year. We know the solar constant is 1367 W m^{-2}. However, at any one time the sun can only illuminates a circle the radius of the earth (πr_{earth}^2). The total area of a sphere with the radius of the earth is ($4\pi r_{earth}^2$). Therefore the average annual amount of energy the earth energy the earth receives from the sun is 1367 / 4 = 341.75 W m^{-2}.

Now let us 'paramaterize' our energy balance model.

Case 1: Atmosphere containing only nitrogen and oxygen (e.g. no greenhouse gases)

- α=0.3
- ε_a = 0.0

The ground temperature T_g = 255° K = -18° C. As we can see, without the warming effect of greenhouse gases our world would be a very cold place.

Case 2: 'Real Earth'

- α=0.3
- ε_a = 0.75

The ground temperature T_g = 287° K = 14° C. The effect of greenhouse gases makes our planet livable.

Case 3: 'Greenhouse world'

- α=0.3
- ε_a = 0.8

The ground temperature T_g = 290° K = 17° C. This is the 'so-called' greenhouse effect – my new orchids would love it!

Case 3: 'Ice-covered world'

- α=0.8
- ε_a = 0.75

The ground temperature T_g = 210° K = -63° C. Brrr!!! We can see that changing albedo is important too.

Convective processes

In addition self-propagating radiation, the earth's land and ocean surfaces exchange energy with the atmosphere through convective processes. We will not consider the physics of these processes in the detail that we covered radiative processes, because the physics are more complicated and fundamental processes of turbulent transfer are not as well understood. I should not that these fluxes occur in what is known as the **boundary layer**.

There are two major convective processes:

1. Sensible Heat Flux (Q_h)

Sensible heat flux occurs if the addition or subtraction of energy from a body is sensed as a change in temperature of that body.

2. Latent Heat Flux (Q_E)

Latent heat transfer occurs if the addition or subtraction of energy from a body enlists a change of state of water (evaporation - liquid to a vapor, condensation – vapor to liquid, or sublimation – solid to vapor). The reason water is so important is that water requires/releases a huge amount of energy when it undergoes a change of state).

Both sensible and latent heat fluxes depend on the ***gradients*** of temperature or amount of water in the atmosphere just above the ground or water surface. The fluxes are also ***directly proportional*** to the wind speed.